The Application of Constraint Semantics to the Language of Subjective Uncertainty

Eric Swanson
ericsw@umich.edu

This is the penultimate version of this paper.
Please consult the official version in the *Journal of Philosophical Logic.*

Abstract: This paper develops a compositional, type-driven constraint semantic theory for a fragment of the language of subjective uncertainty. In the particular application explored here, the interpretation function of constraint semantics yields not propositions but constraints on credal states as the semantic values of declarative sentences. Constraints are richer than propositions in that constraints can straightforwardly represent assessments of the probability that the world is one way rather than another. The richness of constraints helps us model communicative acts in essentially the same way that we model agents' credences. Moreover, supplementing familiar truth-conditional theories of epistemic modals with constraint semantics helps capture contrasts between strong necessity and possibility modals, on the one hand, and weak necessity modals, on the other.

Keywords: Non-truth-conditional theories of meaning · Compositionality · Credence · Assertion · Epistemic modals · Constraint semantics

Both of the following hypotheses have many adherents.

The credal hypothesis: A typical agent's credal state cannot be adequately characterized with a set of propositions alone. For example, no set of propositions would adequately characterize my (purely subjective) 0.5 credence that it rained in Vladivostok yesterday.

The assertion hypothesis: What a typical agent means to communicate can be adequately characterized using propositions alone. For example, what I mean

Thanks to Dave Baker, David Braun, Aaron Bronfman, Nate Charlow, Janice Dowell, Kit Fine, Danny Fox, Dilip Ninan, Paul Portner, David Sobel, Rich Thomason, Tim Williamson, and Steve Yablo for helpful discussion. Thanks also to audiences at Harvard University; University of California, Berkeley; the Formal Epistemology Festival at University of Konstanz; University of Michigan, Ann Arbor; the Chambers Philosophy Conference at University of Nebraska-Lincoln; and Yale University. Thanks especially to Andy Egan, Kai von Fintel, Thony Gillies, Ned Hall, John MacFarlane, David Manley, Sarah Moss, Mark Richard, Robert van Rooij, Daniel Rothschild, Bob Stalnaker, and Seth Yalcin.

to communicate in saying that there is a 50% chance that it rained in Vladivostok yesterday is a proposition about information available to me or to my community.

There is considerable tension between these two hypotheses, for the assertion hypothesis makes it unclear how we could communicate features of our credal states that cannot be characterized with propositions alone.

If you were sympathetic to contemporary work on subjective uncertainty you might favor the credal hypothesis. You might argue that there is no possible worlds proposition such that I lend high credence to that proposition (or believe that proposition) just in case my subjective credence in the proposition that it rained in Vladivostok yesterday is 0.5. And so, you might conclude, we cannot characterize a typical agent's credal state using propositions alone. If you were sympathetic to traditional models of assertion and meaning, on the other hand, you might favor the assertion hypothesis. You might argue that the essential function of assertion is to represent the world as being a certain way, and that propositions, which characterize the meaning of a sentence relative to a context, are no more and no less than the things that can represent any way the world might be (Stalnaker 1978, 1984) or might be thought to be. And you might argue (from a different angle) that the only things that could be the range of a compositional semantic interpretation function with sufficient empirical coverage are truth-conditions, or structured propositions (Frege 1892a, 1892b; Russell 1903; Carnap 1956), or discourse representation structures (Kamp 1981), or context change potentials (Heim 1983), or whatever other non-probabilistic medium you favor.

Section 1 of this paper presents a version of *constraint semantics,* a compositional alternative to traditional truth-conditional semantics. A constraint semantic interpretation function takes a sentence and a context as input and yields not truth-conditions but a constraint as output. Because my focus here is the language of subjective uncertainty, the particular theory I develop here delivers constraints on credal states. Constraints are richer than propositions in that constraints can represent (among many other things) assessments of the probability that the world is one way rather than another. The richness of constraints helps us model communicative acts in ways that cohere naturally with the credal hypothesis. But this richness is at odds with the idea that the essential function of assertion is to represent the world as being a certain way, and so constraint semantics requires some revision of that way of thinking. I suggest that we should instead think of assertion as a way of giving advice. With the language of subjective uncertainty, to assert that ϕ is, very roughly speaking, to advise one's addressees to conform to the constraint that is the image of 'ϕ' under the constraint semantic interpretation function.

Section 2 focuses on two issues concerning epistemic modals. First, it might seem that anyone who abandons the assertion hypothesis in trying to handle the language of subjective uncertainty would also abandon

Kratzer's hypothesis: a given modal has a "common kernel of meaning" whether it is used to target epistemic modality, deontic modality, circumstantial modality, or some other flavor of modality (Kratzer 1977, 338–342; cf. Sloman 1970). That common kernel targets features of a body of information or set of premises.

But I argue that Kratzer's hypothesis is in fact vindicated by a complete constraint semantics for epistemic modals, and helps explain why epistemic modals have their distinctive evidential features. Second, it might seem that only quantitative epistemic hedges *demand* an approach like constraint semantics, and that ordinary epistemic modals can be handled by traditional truth-conditional approaches. But I argue that traditional truth-conditional approaches do not account for important contrasts between weak necessity modals (like 'should' and 'ought') and possibility modals. Blending Kratzer's "common kernel of meaning" together with credal constraint semantics makes it easy to capture the relevant contrasts. So standard treatments of non-quantitative epistemic modals can improved by adding a layer of constraint semantic meaning.

Although this paper focuses on a small fragment of English, I think that constraint semantics can be fruitfully applied to many of the aspects of our doxastic, affective, and conative lives that we communicate to others. I take a narrow focus here for two reasons. First, the language of subjective uncertainty is a good test case for non-truth-conditional approaches to communication. This is because work on the attitudes associated with subjective uncertainty is already relatively rich. We can thus sidestep many issues—like the so-called negation problem (on which see, e.g., Schroeder 2008)—that arise for non-truth-conditional theories of moral discourse. Second, without a fairly detailed theory it would be impossible to assess the value of constraint semantics, and the larger the target linguistic fragment the more unwieldy an appropriately detailed theory would be. There is plenty of interesting work to be done even with our attention largely restricted to epistemic modals, probability operators, and their interactions with connectives and quantifiers.

1. Constraint semantics

In familiar truth-conditional semantics, the semantic interpretation function takes a declarative sentence and a context as input, and returns a set of possible worlds meant to represent the content of that sentence relative to that context. A sentence embedded in a belief ascription is interpreted in the same way. '*S* believes that ϕ' says that S believes the proposition that is the image of 'ϕ' under the semantic interpretation function, relative to the relevant context. The interpretation function of constraint semantics works similarly to the interpretation function of truth-conditional semantics, and plays similar roles. Relative to a context, the constraint semantic interpretation function takes a declarative sentence and returns a constraint meant to represent the content of that sentence relative to that context. And, again relative to a context, 'S believes that ϕ' says that S's credal state has the property represented by the constraint that is the image of 'ϕ' under the semantic interpretation function.[1]

A *constraint* is a set of *admissibles*. Each admissible represents a credal state, and so a constraint represents a property of credal states. Because I am focusing here on the language of subjective uncertainty, I work under the simplifying assumption that constraints and admissibles represent no more than this. But the focus on credal constraints merely keeps the presentation simple; it is easy to extend the notion of constraints as necessary to handle other phenomena. For example, if we want to represent normative language as communicating features of our attitudes that transcend characterization in purely propositional terms, we may extend the notion of constraints so that a given constraint is a set of characterizations of credal states and normative attitudes, such that each characterization is admissible. Similarly, we might say that the interesting content associated with a counterfactual conditional is not a proposition but is instead a constraint on imaging functions, or a special kind of subjective probability. Then we could further extend the notion of a constraint so that the elements of a constraint represent admissible credal states, normative attitudes, and imaging functions or counterfactual probabilities. Extensions like these are easy to add as needed. The complete constraint semantic interpretation function for a language would take *any* declarative sentence of that language and return a constraint that includes a representation of *each* state that is compatible with that sentence and that might be semantically targeted by a declarative sentence.[2]

There is as yet little consensus about what vehicles could give a comprehensive model of a believer's credal states. But to assess the viability of constraint semantics we need to choose a particular vehicle, while bearing in mind that future work in formal epistemology may show this particular vehicle to be inadequate. Fortunately, other vehicles could easily take its place. Here I model credal states using a slight enrichment of probability spaces, where a *probability space* is a triple $\langle W, \mathcal{F}, \mu \rangle$ such that:

1. $\mathcal{F}$ is an algebra over W (i.e., $\mathcal{F}$ is a set of subsets of W, $W \in \mathcal{F}$, and $\mathcal{F}$ is closed under complementation and union);

[1] For similar views, see Swanson 2006, 53 and Schroeder 2008, 136–141.

[2] Another natural extension is to expand the domain of the semantic interpretation function so that it includes some non-declarative sentences.

2. μ is a function from $\mathcal{F} \to [0,1]$;

3. $\mu(W) = 1$;

4. If M and N are disjoint elements of $\mathcal{F}$, then $\mu(M \cup N) = \mu(M) + \mu(N)$.

Note that μ measures the members of $\mathcal{F}$, not the members of W. I henceforth assume that W is the set of possible worlds, and that $\mathcal{F}$ is the power set of W—the set of possible worlds propositions—but for certain purposes we might want to relax these assumptions somewhat and to make revisions elsewhere in the framework as appropriate.[3]

I want to be agnostic about whether belief reduces to credence—and, more broadly, about the relationship between belief and credence—and so I use what I call *enriched probability spaces* to model credal states. In particular, I henceforth let μ (an *enriched probability measure,* or simply a *measure* for short) be a function from $\mathcal{F}$ into $\{\langle x,y\rangle \mid x \in [0,1] \text{ and } y \in \{0,1\}\}$.[4] It will be helpful to simplify the presentation by appealing to Cartesian products, using the notation '$X \times Y$' (where $X \times Y = \{\langle x,y\rangle \mid x \in X \text{ and } y \in Y\}$). Thus we can say that μ is a function from $\mathcal{F}$ into $[0,1] \times \{0,1\}$. The first place of the ordered pairs in the range of μ represents credence; the second place represents belief and disbelief. For example, we represent 0.9 credence in P accompanied by belief in P with the constraint, or set of admissible states, $\{\mu \mid \mu(P) = \langle 0.9, 1\rangle\}$. Of course, it's possible for an agent to neither believe nor disbelieve P; we represent this by including both admissibles according to which P is believed and admissibles according to which P is not believed. (This choice of representation is helpful in certain ways; it simplifies the semantic entry for wide-scope negation.) For example, we represent 0.5 credence in P accompanied by neither belief nor disbelief in P with the constraint $\{\mu \mid \mu(P) = \langle 0.5, 0\rangle \text{ or } \mu(P) = \langle 0.5, 1\rangle\}$ (equivalently, $\{\mu \mid \mu(P) \in \{0.5\} \times \{0,1\}\}$). The probability axioms govern the relationship between the domain of μ and the first place of the ordered pairs in its range. In this representation of disbelief and belief, 0 and 1 are purely conventional, purely arbitrary devices; they are not meant to indicate any similarity with the credences 0 and 1.[5]

[3]For example, epistemic modals are sometimes used to raise possibilities, and we might model this effect by taking $\mathcal{F}$ to be a proper subalgebra of the power set of W. For discussion see Swanson 2006, 77–84 and 2012 and Yalcin 2007 and 2011.

[4]Those who endorse what Richard Foley calls the "Lockean thesis" about belief might replace enriched probability measures with ordinary probability measures (making appropriate tweaks here and there) in giving a constraint semantics for the language of subjective uncertainty. According to the Lockean thesis, "it is epistemically rational for us to believe a proposition just in case it is epistemically rational for us to have sufficiently high degree of confidence in it, sufficiently high to make our attitude towards it one of belief" (Foley 1992, 111).

[5]Timothy Williamson has argued both for a distinction between belief and credence and for the

On this approach, a constraint on credal states is just a set of enriched probability measures. As a prolegomenon to doing compositional constraint semantics, a good heuristic for determining what credal constraint *should* be associated with the sentence 'ϕ' is to ask what is distinctively associated with believing that ϕ.[6] For example, the constraint that should be associated with (1) is the set of measures that take the proposition that it rained in Vladivostok yesterday to an element of $[0, 1] \times \{1\}$.

(1) It rained in Vladivostok yesterday.

Similarly, what is distinctively associated with believing that there's a 50% chance that it rained in Vladivostok yesterday is lending 0.5 credence to the proposition that it rained in Vladivostok yesterday.[7] So the constraint that should be associated with (2) is the set of measures that take the proposition that it rained in Vladivostok yesterday to an element of $\{0.5\} \times \{0, 1\}$; i.e., to $\langle 0.5, 0\rangle$ or to $\langle 0.5, 1\rangle$.

(2) There's a 50% chance that it rained in Vladivostok yesterday.

And the constraint that should be associated with (3) is the set of measures that take the proposition that the last ball drawn was white to $\{0.25\} \times \{0, 1\}$ and the proposition that the last ball drawn was red to $\{0.25\} \times \{0, 1\}$.

(3) There's a 25% chance that the last ball drawn was white, and a 25% chance that the last ball drawn was red.

There are some formal similarities between my representation of constraints and certain representations of "imprecise credences." But the interpretation of the formalisms is very different. Each enriched probability measure in a constraint represents a particular credal state with a great degree of precision. Theorists who hold that a believer's credal state can be imprecise[8] might well say that this level of precision is inappropriate to the task, and favor a different kind of vehicle for the representation of admissibles. Such theorists obviously would need to tell a somewhat more complicated story about the relationship between believers and constraints: each admissible might be a set of measures, for example, or a set of measures and a measure over them.

---

hypothesis that "Outright belief still comes in degrees" (2000, 99). It is not clear that the phenomena Williamson adduces in support of the latter hypothesis should be represented in semantics; for simplicity I suppose that they should not.

[6] As SCHROEDER 2008 nicely puts a related thought: "The expressivist strategy is to explain the language in some domain *by* explaining the thought in that domain" (152). See also GIBBARD 1990: "the meaning of normative terms is to be given by saying what judgments normative statements express—what states of mind they express" (84).

[7] For simplicity I write as though we are always extremely precise about credences; this assumption could easily be dropped.

[8] E.g., VAN FRAASSEN 1980, LEVI 1980 and 1985, JEFFREY 1983, WALLEY 1991, KAPLAN 1996, JOYCE 2005, and STURGEON 2008.

Bracketing such complications makes constraint semantics easier to understand and to assess on its own terms.

It is not hard to give a constraint semantics adequate to cases like (1)–(3). The implementation I lay out here gives proper names their standard semantic type and semantic values—'Al,' for example, denotes Al in all possible worlds—but it changes the type and semantic values of predicates, operators, connectives, and quantifiers. In ordinary truth-conditional semantics, the semantic value of 'is tall' is a function that takes (for example) the denotation of a name and returns the proposition consisting of those worlds in which the actual denotation of the name is tall. Relative to a world of evaluation, the value of this function when applied to Al is *true* iff Al is tall in that world. Constraint semantics still makes use of *true* and *false,* but merely to characterize sets in the familiar way—any other pair of objects could serve the same purpose. After all, the ultimate goal is not for the semantic value of a sentence to be a possible worlds proposition, but for it to be a constraint. Accordingly, the semantic value of 'is tall' is a function that takes (for example) the semantic value of a name and returns a constraint—namely, the constraint that is the set of measures that take the proposition that the actual bearer of that name is tall to an element of $[0, 1] \times \{1\}$. (This is the set of measures that represents belief in the proposition that the actual bearer of that name is tall, and that does not represent any particular credence in that proposition.) So when we combine the name 'Al' with the predicate 'is tall' in the sentence 'Al is tall,' we get the set of measures that take the proposition that Al is tall to an element of $[0, 1] \times \{1\}$. The semantic value of an operator like 'there's a 50% chance that' is (on a first pass) the smallest function that takes the set of all and only those measures that take some proposition to an element of $[0, 1] \times \{1\}$, and returns the set of all and only those measures that take that proposition to an element of $\{0.5\} \times \{0, 1\}$. So when we combine this operator with the clause 'Al is tall,' in the sentence 'There's a 50% chance that Al is tall,' the constraint semantic interpretation function returns the set of measures that take the proposition that Al is tall to an element of $\{0.5\} \times \{0, 1\}$. (This is the set of measures that represents credence of 0.5 in the proposition that Al is tall, and is neutral with respect to belief in that proposition.)

Spelling this out more formally, and with a bit more detail, will bring out important similarities between constraint semantics and traditional truth-conditional semantics. There are four basic types: the type of individuals, the type of possible worlds, the type of truth values—again, used only to characterize sets—and the type of admissibles.

e is the type of individuals; its domain is $\{\text{Al, Betty}\}$.
s is the type of possible worlds; its domain is W (the set of possible worlds).
t is the type of truth values; its domain is $\{\mathit{true}, \mathit{false}\}$.
a is the type of admissibles; its domain is the set of all enriched probability measures.

If α and β are types, then $\langle \alpha, \beta \rangle$ (sometimes abbreviated $\alpha\beta$) is a type.
Nothing else is a type.

Other types are constructed out of these in the familiar way. For example, constraints are type $\langle a, t \rangle$—functions that take an admissible and return a truth value—because a constraint is a set of admissibles.[9]

Again, proper names are treated as usual. As rigid designators, their semantic values are constant intensions. The semantic value of a proper name is a function that takes a possible world and returns the individual that the name denotes in the actual world. (Subscripts indicate semantic type.)

$[\![\textbf{Al}]\!] = \lambda w_{\langle s \rangle}.\text{Al}$

$[\![\textbf{Betty}]\!] = \lambda w_{\langle s \rangle}.\text{Betty}$

But an ordinary predicate is a function that takes an individual concept—a type $\langle s, e \rangle$ expression, like a name—and returns not a proposition but a constraint:

$$[\![\textbf{is tall}]\!] = \lambda N_{\langle s,e \rangle}.\lambda a. \begin{cases} \textit{true} \text{ if } a \text{ takes the proposition } \{w_{\langle s \rangle} \mid N(w) \text{ is tall in } w\} \\ \text{to an element of } [0,1] \times \{1\}; \\ \textit{false} \text{ otherwise.} \end{cases}$$

In (post-Carnapian) English, this says that the semantic value of 'is tall' is a function that takes an individual concept N and returns a function that takes admissibles to truth values. In particular, that function takes an admissible a to *true* iff a takes to an element of $[0,1] \times \{1\}$ the set of those worlds w in which the extension of N at w is tall in w. The function takes admissibles that do not meet this condition to *false*. The semantic value of 'is nice' works in just the same way. Put a little differently, it takes an individual concept N and returns a function that takes an admissible a to *true* iff a takes the proposition that, for any world w, the extension of N in w is nice in w to an element of $[0,1] \times \{1\}$.

$$[\![\textbf{is nice}]\!] = \lambda N_{\langle s,e \rangle}.\lambda a. \begin{cases} \textit{true} \text{ if } a \text{ takes the proposition } \{w_{\langle s \rangle} \mid N(w) \text{ is nice in } w\} \\ \text{to an element of } [0,1] \times \{1\}; \\ \textit{false} \text{ otherwise.} \end{cases}$$

Because 'Al' and 'Betty' are rigid designators, the semantic value of (for example)

[9]For similar ways of thinking about semantic value see SWANSON 2006, YALCIN 2007 and 2010, and Moss forthcoming. Although my semantics is in some ways more complicated than Yalcin's, the complications are necessary to handle the "third grade of [epistemic] modal involvement" (QUINE 1953): quantifiers that scope over epistemic modals and other hedges.

(4) Al is nice.

will simply be the set of admissibles that take the proposition that Al is nice to an element of $[0, 1] \times \{1\}$. Similarly, the semantic value of

(5) Betty is tall.

will be the set of admissibles that take the proposition that Betty is tall to an element of $[0, 1] \times \{1\}$.

Similar sentences with non-rigid definite descriptions in the subject position will generate slightly more complicated constraints. Obviously the extension of 'the tallest person' varies from world to world: even if Betty is in fact the tallest person, she is not the tallest person as a matter of necessity. So the semantic values of (5) and (6) should not be the same.

(6) The tallest person is tall.

To get appropriate variation in the extension of 'the tallest person,' we can simply give a semantic entry like this one:

$[\![$**the tallest person**$]\!] = \lambda w.x$ such that x exists in w and x is a taller person than any other person in w. (Cf. Thomason & Stalnaker 1968, Thomason 1969, and Heim 1991.)

The semantic value of (6) will then be the set of admissibles that take the proposition that is true at a world w iff the tallest person in w is tall in w to an element of $[0, 1] \times \{1\}$. If Al is the tallest person in some world w', and Al is tall in w', then the proposition taken to an element of $[0, 1] \times \{1\}$ will include w'. Similarly, if Betty is the tallest person in some world w'', and Betty is tall in w'', then the proposition taken to an element of $[0, 1] \times \{1\}$ will include w''.[10]

We can now move on to the language of subjective uncertainty. Here is a first pass at a semantic entry for an explicitly quantitative sentential operator:

$[\![$**there is an $x\%$ chance that**$]\!] =$

[10] To be sure, many interesting constructions are beyond the ambit of this implementation of constraint semantics. For example, consider

(4) Few likely winners are tall.

To handle cases like (4) we would need noun phrases to be of a higher type. (For example, we might make noun phrases type $\langle\langle se, at\rangle, at\rangle$ and keep predicates type $\langle se, at\rangle$; alternatively we might raise both.) Thanks to Daniel Rothschild for questions here.

$$\lambda C_{\langle a,t\rangle}.\lambda a.\begin{cases} \textit{true} \text{ if, for all propositions } P \text{ such that every admissible of } C \\ \text{takes } P \text{ to an element of } [0,1] \times \{1\}, \\ a \text{ takes } P \text{ to an element of } \{\frac{x}{100}\} \times \{0,1\}; \\ \textit{false} \text{ otherwise.} \end{cases}$$

For example, consider a substitution instance of this hedge scoped over a sentence like (6):

(8) There is a 50% chance that Betty is tall.

The semantic value associated with (8) will be the set of admissibles that take the proposition that Betty is tall to an element of $\{0.5\} \times \{0,1\}$; i.e., to $\langle 0.5, 0\rangle$ or to $\langle 0.5, 1\rangle$.[11]

As successful as it is here, this semantic entry for 'there is an x% chance that' oversimplifies. Call a clause *unqualified* just in case there is a proposition taken to an element of $[0,1] \times \{1\}$ by every admissible in the constraint associated with the clause; call other clauses *qualified*. (Note that the language of subjective uncertainty can be found in unqualified clauses. For example, "Betty believes that it might rain" and "We're going to call everyone who might be lost" are presumably both unqualified clauses. The role that the language of subjective uncertainty plays in relative clauses is fascinating, but I have to leave it to future work.) This semantic entry oversimplifies in virtue of mishandling cases where 'there is an x% chance that' scopes over an qualified clause. Here is an example. On the constraint semantic approach, (9), read as an expression of subjective uncertainty, is treated as a qualified clause. The constraint associated with it includes only admissibles on which the credence assigned to the proposition that it rains is twice that assigned to the proposition that it snows.

(9) It is twice as likely to rain as it is to snow.

But clearly (9) can be embedded under 'there is an x% chance that,' as in

(10) There is a 50% chance that it is twice as likely to rain as it is to snow.

And the semantic entry mooted above for 'there is a 50% chance that' simply would not engage with the semantic value of (9).

This does not show that there is a problem with constraint semantics per se—it shows only that the above "first pass" semantic entry for 'there is an x% chance that' is not the final story. That semantic entry, oversimple as it is, does constitute a pointer

[11](8) also has a reading on which its semantic value is the set of admissibles that take the proposition that Betty is tall to an element of $[0.5, 1] \times \{0,1\}$; adding wide-scope negation helps make this reading salient. For example, 'There isn't a 50% chance that Betty is tall' can be read as saying that there is a less than 50% chance that Betty is tall. Thanks to Daniel Rothschild for these observations.

toward the final story: it gives the right results in the cases in which 'there is an $x\%$ chance that' scopes over a unqualified clause. Ultimately I would be delighted to have a constraint semantics that gave the right results in other cases as well.

But the next step in the process of developing such a theory is not doing more semantics but figuring out how to model the doxastic states associated with claims like (10). After such work is done, the representation of the relevant doxastic states can be written into constraint semantics. I won't try to do that kind of work here; I am simply trying to make it plausible that the results of the intermediate step can find a home in a complete semantic theory.

Now let us turn to connectives. The semantic entry for sentential 'and' is very straightforward; the constraint associated with 'ϕ and ψ' includes all and only the admissibles associated with *both* 'ϕ' and 'ψ.'

$$[\![\textbf{and}]\!] = \lambda F_{\langle a,t\rangle}.\lambda G_{\langle a,t\rangle}.F \cap G$$

For example, the constraint semantic value of (11) is the set of enriched probability measures that take the proposition that Al is tall to an element of $\{0.5\} \times \{0, 1\}$. And the constraint semantic value of (12) is the set of measures that take the proposition that Betty is nice to an element of $\{0.6\} \times \{0, 1\}$.

(11) There is a 50% chance that Al is tall.

(12) There is a 60% chance that Betty is nice.

From these facts, and the semantic entry for 'and,' the constraint semantic value of

(13) There is a 50% chance that Al is tall and there is a 60% chance that Betty is nice.

is the set of enriched probability measures that assign the credence 0.5 to the proposition that Al is tall *and* assign the credence 0.6 to the proposition that Betty is nice. *S* believes that there is a 50% chance that Al is tall and there is a 60% chance that Betty is nice just in case *S*'s credal state has the property represented by the associated constraint. So *S* believes that there is a 50% chance that Al is tall and there is a 60% chance that Betty is nice just in case *S* lends credence 0.5 to the proposition that Al is tall, and lends credence 0.6 to the proposition that Betty is nice.

In a truth-conditional framework, the semantic value of sentential negation takes a proposition and returns the proposition that is its complement. At a high level of abstraction, we might say that in a truth-conditional framework, negation targets content. That sort of approach doesn't generalize to constraint semantics. Roughly put, constraint semantic sentential negation leaves the content intact and instead targets the attitude taken toward that content. More precisely,

$$[\![\textbf{not}]\!] = \lambda F_{\langle a,t\rangle}.\lambda a.\begin{cases} \textit{true} \text{ if } F(a) = \textit{false};\\ \textit{false} \text{ otherwise.}\end{cases}$$

For example, recall that the constraint semantic value of

(12) There is a 60% chance that Betty is nice.

is the set of measures that take the proposition that Betty is nice to an element of $\{0.6\} \times \{0,1\}$. The constraint semantic value of (14), then, is the set of measures that take the proposition that Betty is nice to an element of $[0,1]\backslash\{.6\} \times \{0,1\}$.

(14) It is not the case that there is a 60% chance that Betty is nice.

In other words, the constraint semantic value of (14) is the set of enriched probability measures that do not assign 0.6 to the proposition that Betty is nice. Here is an example of negation over an unqualified clause. Recall that the semantic value of

(5) Betty is tall.

is the set of measures that take the proposition that Betty is tall to an element of $[0,1] \times \{1\}$. Adding negation yields (15), the constraint semantic value of which is the set of measures that take the proposition that Betty is tall to an element of $[0,1] \times \{0\}$.

(15) Betty is not tall.

The constraint associated with (15), then, represents disbelief in the proposition that Betty is tall.

For familiar reasons it is standard to associate conjunction with set-theoretic intersection and disjunction with set-theoretic union. When each disjunct of a disjunction can be modeled adequately using a possible worlds proposition, belief in that disjunction just is belief in the proposition that is the union of the propositions expressed by the disjuncts. The semantic entry for 'and' that I gave earlier is in this Boolean mold: it takes a pair of constraints and returns their intersection. But the constraint that should be associated with a disjunction cannot, in general, be the union of the constraints associated with each of the disjunction's disjuncts. For example, a believer may believe a disjunction—'Probably ϕ or probably ψ,' say—without believing any particular disjunct. If the constraint associated with a disjunction were the union of the constraints associated with its disjuncts, this would be impossible. And so De Morgan's laws are invalid for the language of subjective uncertainty: if they were valid, then the simple Boolean entries for negation and conjunction would commit us to a badly implausible semantics for disjunction.

What kind of constraint semantic entry *should* we give for disjunction? Recall the rough heuristic I mentioned earlier: to determine what constraint to associate with the sentence 'ϕ,' ask what is distinctively associated with believing that ϕ. Although we can say some things about the kinds of belief state that can and can't count as believing a disjunction involving the language of subjective uncertainty, precise characterizations prove elusive. To see this it will help to look at some popular characterizations of rational credence change.

Consider, for example, simple conditionalization and Jeffrey conditionalization (Jeffrey 1968). These procedures will not help us characterize the state of believing a disjunction, since cases in which a disjunct is qualified—like (16)—do not supply a proposition to conditionalize on or to Jeffrey conditionalize on.

(16) We're either as likely as not to hire John, or we're as likely as not to hire James—you know how bad I am with names.

We do no better with 'infomin' or 'maxent,'[12] construed as procedures on which a believer who learns a disjunction minimizes relative entropy between the prior and posterior, while ensuring that the posterior conform to constraints associated with the disjunction. Among other things, this kind of procedure raises a variation of Bas van Fraassen's 'Judy Benjamin' problem. In its original formulation:

> The war games area is divided into the region of the Blue Army, to which Judy Benjamin and her fellow soldiers belong, and that of the Red Army. Each of these regions is further divided into Headquarters Company Area and Second Company Area. The patrol has a map which none of them understands, and they are soon hopelessly lost. Using their radio they are at one point able to contact their own headquarters. After describing whatever they remember of their movements, they are told by the duty officer 'I don't know whether or not you have strayed into Red Army territory. But if you have, the probability is $\frac{3}{4}$ that you are in their Headquarters Company Area.' At this point the radio gives out.…
>
> *Question:* What will be Private Benjamin's posterior probability that she is in the friendly Blue Army Region? (1981, 377; see also his 1989, 342 ff.)

As van Fraassen et al. 1986 observe, "In audience surveys in classes and at lectures, the overwhelming preliminary reaction is that the probability of Blue should stay the same." But they argue that "neither infomin, nor any other plausible rule agrees; and

[12] On which see (e.g.) Kullback & Leibler 1951, Jaynes 1957, Williams 1980, and van Fraassen 1980.

no acceptable rule could give that reply in the general case" (455).[13] In its disjunctive reformulation, the problem is as before, except

> ...they are told by the duty officer 'I don't know whether or not you have strayed into Red Army territory. You're either in Blue Army territory or the probability is $\frac{3}{4}$ that you are in the Red Army Headquarters Company Area.' At this point the radio gives out ....
>
> *Question:* What will be Private Benjamin's posterior probability that she is in the friendly Blue Army Region?

Suppose we ask that the posterior satisfy the (natural looking) constraint that it's three times likelier that Judy Benjamin is in Red Army Headquarters Company Area than that she is in Red Army Second Company Area. Then we get exactly the same counterintuitive results as in van Fraassen's original example: Judy Benjamin's credence that she is in Blue Army territory should increase from 0.5 to approximately 0.533. Whether or not we can find a constraint that gives intuitively acceptable results for the disjunctive variant on van Fraassen's example, we are far from having a fully general characterization of disjunctive belief. And so we do not have a satisfactory characterization of what it is to believe a disjunction when at least one disjunct involves the language of subjective uncertainty.

A complete theory of subjective uncertainty itself must include a characterization of the function from the constraints associated with disjuncts to the constraint associated with their disjunction. But if we restrict our theorizing to the *language* of subjective uncertainty—as I am doing here—we do not need a detailed answer to this question.[14] It is legitimate for a semantic theory to help itself to elements of a theory of belief. So here I simply help myself to a function that provides what I call *disjunctive mixtures.* The disjunctive mixture of a set of constraints is the constraint associated with the disjunction of sentences that express those constraints.[15] The *disjunctive mixture function* takes a set of constraints and returns its disjunctive mixture. The semantic entry for sentential two-place 'or' can then be given in terms of the disjunctive mixture function.

$$[\![\mathbf{or}]\!] = \lambda F_{\langle a,t\rangle}.\lambda G_{\langle a,t\rangle}.\lambda a. \begin{cases} \textit{true} \text{ if } a \text{ is a member of the set that is the image} \\ \text{of } \{F, G\} \text{ under the disjunctive mixture function;} \\ \textit{false} \text{ otherwise.} \end{cases}$$

[13]For a nice generalization of van Fraassen's example, see Seidenfeld 1986.

[14]Moreover, it's possible that our final account of disjunction should be informed by other kinds of non-truth-conditional language as well. See especially the discussions of disjunction in Gibbard 2003 and Schroeder 2008.

[15]The talk of 'mixtures' is meant only as a heuristic: perhaps disjunctive mixtures should be represented using sets of probability spaces, in a way that forgoes anything like 'mixture.'

As I said earlier, the hypothesis that enriched probability spaces adequately represent believers' credal states is provisional. Future research may well show that it is false. The current discussion makes the status of this hypothesis especially salient, however, because it's possible that we will need a different representation of credal states to characterize the doxastic states associated with believing a disjunction. It is obviously hard to be sure exactly what theoretical devices the solution to an open problem might require. But suppose, to take just one example, that we need to model a doxastic state using an evolution rule over enriched probability measures that is not itself determined by an enriched probability measure. It might then be tempting to hold fixed the semantic values of unqualified clauses while saying that the semantic value of a disjunction of unqualified clauses is of a higher type. For example, the semantic value of a disjunction of unqualified clauses could be a function from an enriched probability measure to an enriched probability measure. But to take this course would be in effect to abandon the goal of giving a relatively straightforward compositional semantics for the language of subjective uncertainty.[16]

A better course, I think, is to enrich admissibles across the board. Like the semantic value of an unqualified clause, the semantic value of a disjunction should be a set of admissibles. If we learn that the credal state associated with believing a disjunction requires a richer representational vehicle than enriched probability measures, then we should take that not just as something we have learned about the semantic value of disjunctions, but as something we have learned about *admissibles in general.* In other words, we should generalize to the worst case, employing the same richer representation for all clauses (cf. Montague 1973). The semantic entries that I have given here would have to be revised somewhat to accommodate this sort of change, but the necessary revisions are straightforward. To take one simple example, we could easily add an evolution rule or distance measure defined over enriched probability measures to the semantic values used here. We would simply move from treating admissibles as ordered pairs representing credence and belief to treating admissibles as ordered triples representing credence, belief, and the relevant evolution rule or distance measure.

The need for this kind of revision should be neither worrying nor surprising. Constraint semantics is attractive in large part because it provides a *framework* for theorizing about natural language meaning that can assimilate and deploy representations of credal states that are richer than truth-conditions. But because it is not yet clear what representation of credal states is adequate, it is not yet clear exactly what innovations semanticists should adopt. The right way to develop constraint semantics is thus to adopt provisional representations of credal states that are helpful for present purposes. While doing this we should be careful not to put unnecessary

[16] Thanks to Daniel Rothschild for pressing this point.

weight on provisional parts of our theories, and careful to explain what is inessential about those provisional parts. We can thus do compositional semantics while leaving problems having to do with the representation of credal states to formal epistemology. This divide-and-conquer strategy is productive not only because it allows semanticists to put helpful technology to use, but also because it lets us theorize about the representation of credal states separately from the compositional derivation of credal constraints.

I think that a similar strategy should be pursued with conditionals: we should not occupy semantics with tasks that are the responsibility of formal epistemology. It is fairly clear what sort of constraint should be associated with

(17) If there was any precipitation last night, it was snow.

The relevant sort of constraint should make the conditional probability of snow on precipitation 1 (or perhaps should make it 'close enough' to 1). It is straightforward to formulate a constraint semantic entry for 'if' that will deliver this result. But it is famously hard to say what credal state or states should be associated with

(18) If Kripke was there if Strawson was, then Anscombe was there. (Gibbard 1981, 235)

(Although Gibbard claims that "Many embeddings of indicative conditionals …seem not to make sense," (18) is intelligible in some contexts.) The difficulties in saying what constraint should be associated with (18) reflect the fact that the final semantic entry for 'if' in (18) must traffic in more than the conditional probability of one proposition on another. But, again, this not a problem for constraint semantics. It is a problem in formal epistemology: a problem of characterizing the relationship between the credal states that we can target using conditionals and the credal states associated with the antecedents and consequents of those conditionals. Constraint semantics is sufficiently catholic about the nature of constraints that once we have that characterization it will be straightforward to deploy it in a constraint semantic entry for 'if.'

By contrast, semantic theory *is* responsible for the interaction between quantifiers and modal operators. Consider the following cases, in which quantifiers scope over epistemic modals:

(19) "Every moment you spend with your child could be the one that really matters." (Russert 2006, xv–xvi)

(20) Given only what we can be certain of, no one here has to be the thief.

(21) I was careful, but I was painting the ceiling, and you know how that can be. Almost every square inch of the floor might have paint on it.

It's crucial to capturing the meaning of these sentences that the quantifier scopes over the epistemic modal. For example, (19) obviously cannot be paraphrased by the anomalous

(22) #It could be that every moment you spend with your child is the one that really matters.

And the second sentence of (21) obviously does not mean what (23) does, with its epistemic 'might' taking wide scope.

(23) It might be that almost every square inch of the floor has paint on it.

As Swanson 2010a argues, traditional force modifier accounts—with their sharp separation of force and content (Dummett 1981, 330)—cannot handle examples like (19)–(21).

In standard truth-conditional semantics, the semantic values of quantifier phrases can be modeled as being of type $\langle\langle se, st\rangle, st\rangle$. That is, the semantic value of a quantifier phrase is a function that takes an intension associated with a one-place predicate (type $\langle se, st\rangle$) and returns a proposition (type $\langle s, t\rangle$). In keeping with the semantic values characteristic of constraint semantics, we replace type $\langle s, t\rangle$ throughout the semantics with type $\langle a, t\rangle$. In constraint semantics, then, the semantic values of quantifier phrases are type $\langle\langle se, at\rangle, at\rangle$.

To understand how the semantic values work it will be helpful to look briefly at quantifier phrases in standard truth-conditional semantics. Here are some traditional truth-conditional semantic entries:

$$[\![\textbf{some person}]\!] = \lambda P_{\langle se,st\rangle}.\lambda s.\begin{cases} \textit{true} \text{ if for some person } e \text{ in } s, \\ \big(P(\langle s, e\rangle)\big)(s) = \textit{true}; \\ \textit{false} \text{ otherwise.} \end{cases}$$

$$[\![\textbf{most people}]\!] = \lambda P_{\langle se,st\rangle}.\lambda s.\begin{cases} \textit{true} \text{ if for most people } e \text{ in } s, \\ \big(P(\langle s, e\rangle)\big)(s) = \textit{true}; \\ \textit{false} \text{ otherwise.} \end{cases}$$

$$[\![\textbf{exactly five people}]\!] = \lambda P_{\langle se,st\rangle}.\lambda s.\begin{cases} \textit{true} \text{ if for exactly five people } e \text{ in } s, \\ \big(P(\langle s, e\rangle)\big)(s) = \textit{true}; \\ \textit{false} \text{ otherwise.} \end{cases}$$

This sort of approach gives us a relatively familiar way to think about predication with quantifier phrases. But we could think about this sort of predication in other ways.

For example, we could think of it as a disjunction distributed over sets of individual concepts.

Here is a way to spell out that thought. Let $\mathcal{M} = \{M \mid M$ is a set of type $\langle s, e\rangle$ objects, and for any given world w, the set consisting of the images of w under the members of M includes most people in $w\}$. We can now give a semantic entry for 'Most people are tall,' for example, that treats it as a disjunction over the members of $\mathcal{M}$, each disjunct of which says that all the members of some set in $\mathcal{M}$ are tall: $\bigvee_{M\in\mathcal{M}} \forall x(x \in M \to x \text{ is tall})$. In terms of a semantic entry for the generalized quantifier itself:

$$[\![\textbf{most people}]\!] = \lambda P_{\langle se,st\rangle}.\bigcup_{M\in\mathcal{M}}\bigcap_{i\in M} P(i)$$

The idea behind this approach can be adapted to constraint semantic entries that successfully handle examples like (19)–(21). 'Most people' takes the semantic value of a predicate P and returns the disjunctive mixture of a set of constraints. The members of that set of constraints are, individually, the result of conjoining the constraints yielded by applying P to the elements of a set in $\mathcal{M}$. Again, in terms of a semantic entry for 'most people' itself:

$$[\![\textbf{most people}]\!] = \lambda P_{\langle se,at\rangle}.\bigvee_{M\in\mathcal{M}}\bigcap_{i\in M} P(i)$$

(I have written the disjunctive mixture function as '$\bigvee$' to help bring out the parallels between these two entries.) This basic strategy works for any quantifier.[17] So the constraint semantic value of

(21) Almost every square inch of the floor might have paint on it.

is the image under the disjunctive mixture function of a set of constraints. Any given constraint in that set is the constraint associated with the claim that, for a set of square inches on the floor consisting of almost every such square inch, every square inch in that set might have paint on it. We thereby analyze the quantifier of (23) as taking widest scope.

[17]Of course, each quantifier needs to be associated with a set of sets of individual concepts, but the recipe is straightforward: the relevant set for a quantifier phrase 'QP' will be $\{Q \mid Q$ is a set of type $\langle s, e\rangle$ objects, and for any given world w, the set consisting of the images of w under the members of Q includes QP in $w\}$. For example, for 'some person'/'exactly five people' we take $\{Q \mid Q$ is a set of type $\langle s, e\rangle$ objects, and for any given world w, the set consisting of the images of w under the members of Q includes some person/exactly five people in $w\}$. It might seem odd to associate a quantifier phrase like 'every person' with a disjunction, but because the conjunctions that make up the disjuncts each target every person, the whole disjunction is equivalent to a conjunction of claims targeting each person.

Associating constraint semantic values with the language of subjective uncertainty is only part of the story; we also need to say what speakers *do* with the language of subjective uncertainty. Propositions represent ways the world might be; constraints generally do not. The difference between the constraint that takes the proposition that it rained yesterday in Vladivostok to 0.5 and the constraint that takes that proposition to 0.6 needn't correspond to any difference in the world. So the move away from truth-conditional semantics to constraint semantics is also a move away from a way of doing semantics that is amenable to treating assertion as a kind of representation.

Here is a first, unsuccessful pass at saying what we do, on the constraint semantic picture, when we assert that ϕ:

Constraint semantics assertion (first pass): In asserting that ϕ, a speaker advises her addressees to conform their doxastic state to the semantic value of 'ϕ'.

This answer fails because it neglects differences between the norms that govern uses of (for example) 'Might ϕ,' 'Must ϕ,' and 'ϕ' simpliciter. In particular, it does not accommodate the very modest intentions and aspirations that speakers often have when they use epistemically hedged statements. Suppose that I do not know where my car keys are, and neither does my friend; he does not know where I've looked; he says

(24) Your keys might be on the kitchen table.

In many cases he will have spoken appropriately even if I have already searched the kitchen table and know that my keys are not there. I can't criticize him for giving bad credal advice. So my friend intends his advice to have *no force* if I already know that the keys are not on the table. He is advising only that I not *inadvertently* rule out or overlook the possibility that my keys are on the kitchen table.[18]

If we were willing to forgo the hope of a unified theory, we might try

Constraint semantics assertion (second pass): In asserting that it might be that ϕ, a speaker defeasibly suggests that her addressees conform their credences to the semantic value of 'Might ϕ'.

The shift from 'advises her addressees to conform' to 'defeasibly suggests that her addressees conform' would likely help explain the norms for the use of 'might.' And we certainly could go on to give further clauses for other epistemic modals and adjectives. But such a theory would miss some important generalizations connecting

[18] For discussion of some similar examples, see von Fintel & Gillies 2005, 10, 14; 2008; and 2011; Swanson 2006, 40-42; and 2011a; Dowell 2011; and MacFarlane 2011.

the language of subjective uncertainty to the norms governing its use. In particular, such a theory would leave unexplained the relationship between speakers' use of the language of subjective uncertainty and the epistemic authority that they portray themselves as having.

To be sure, *many* factors influence conversational participants' assessments of the speaker's epistemic authority with respect to a given subject matter. For example, in coming to a view about the speaker's authority conversational participants often consider the speaker's epistemic position, track record, apparent sincerity and cooperativeness, values, impairments, and so on. But the speaker's use of the language of subjective uncertainty can play an important role, too: speakers use such language to signal that they *claim* less authority than they otherwise might be thought to claim. For example, a speaker who says that the keys might be on the kitchen table is, intuitively, claiming less epistemic authority than a speaker who says that the keys are on the kitchen table.

This phenomenon resembles the kind of conversational implicature often known as 'quantity' or 'scalar' implicature in many respects. For example, when someone says

(25) I ate some of the leftovers.

we are disposed to infer that it is not the case that she believes that she ate all of the leftovers. Although it's not entirely clear how to spell out the details,[19] this inference is licensed at least in part by the fact that a speaker who who says (25) could easily have said, instead, the more informative:

(26) I ate all of the leftovers.

The fact that the speaker said (26) instead of the more informative (25) suggests that she does not take herself to be in a position to assert (26).

Consider now a speaker who says

(24) Your keys might be on the kitchen table.

A speaker who chooses to say (26) instead of (27), (28), or (29) suggests that she is not in a position to assert any of these (intuitively) more informative claims:

(27) Your keys are probably on the kitchen table.

(28) Your keys are almost definitely on the kitchen table.

(29) Your keys are on the kitchen table.

[19] Grice 1987, 26 is the *locus classicus.* See Chapman 2005, 87–94 for an illuminating discussion of Grice's historical context. See Swanson 2010b and the citations therein for some details on the derivations of such implicatures.

In many contexts, such a choice will give addressees reason to infer that the speaker is not well positioned to say any of (27)–(29), because a speaker who *was* well positioned to say one of those and chose to say (24) instead would be uncooperative or irrational. So a speaker who is believed to be largely cooperative and rational and chooses to say (24) generally portrays herself as claiming less epistemic authority than a speaker who says one of (27)–(29). Similarly for more informative claims: a speaker who chooses to say (27), for example, generally portrays herself as claiming less epistemic authority than a speaker who says (28) or (29).

Data and explanations like these lend support to the following generalization.

Constraint semantics assertion (final pass): In asserting that ϕ, a speaker advises her addressees to conform their credences to the semantic value of 'ϕ'. Holding all other factors fixed, the advice associated with 'ϕ' is *stronger* than the advice associated with 'ψ' iff 'ϕ' is more informative than 'ψ'.

Differences in the strength of advice help us explain why we bring different standards to bear on the evaluation of claims that are hedged to different degrees. We hold relatively strong advice to higher standards than relatively weak advice. 'It might be that ϕ' is relatively weak advice, because it admits such a wide range of credence assignments that by saying it a cooperative speaker signals that she does not have the epistemic authority to say anything that is particularly committal about whether ϕ. This helps explain why my friend's suggestion that the keys might be on the table is not criticizable in the ways that unqualified assertions are.[20]

2. Epistemic modals

It has been widely accepted since at least Karttunen 1972 that a sentence headed by English epistemic 'must' typically signals that the content of its prejacent is the conclusion of an inference, not a deliverance of the speaker's direct experience (Coates 1983, 41, 131, 177; Palmer 2001, 34–35). G. E. Moore noticed this feature of epistemic 'must' even earlier:

> 'You *must* have omitted to turn the light off' means: 'There's conclusive evidence that you didn't.' The evidence is: It wouldn't have been on now, if you had turned it off, for (a) nobody else has been in the room & (b) switches can't turn on by themselves. But 'you certainly didn't' doesn't = 'You *must* have omitted': we shouldn't say the latter if we *saw* you come out without turning it off: we then shouldn't have *inferred* that you didn't. (1962, 188, dating to 1941 or 1942)

[20] For further discussion see section 4 of Swanson 2011a.

The epistemic reading of (30), for example, signals that from information that is in some sense available to the speaker it can be inferred that John is here now.

(30) John must be here by now.

Similar signals are often carried by other English epistemic modals. Consider the epistemic readings of (31)–(33)[21]:

(31) John has to be here by now.

(32) John should be here by now.

(33) John ought to be here by now.

Again, when a speaker uses one of these she generally signals that it follows from information that is available to her that John is here now. Without taking a stand on the relationship between epistemic modals and evidentials proper (on which see Speas 2008) I will call this the *evidential feature* of English epistemic modals. English possibility modals in their epistemic readings have a very similar feature, but it is hard to see it when they are not in the scope of negation.

(34) John couldn't be here by now.

(34) suggests not that the speaker has seen that John is not here now, but (again) that John's not being here now follows from some information that is available to her.

The systematicity of the evidential feature of English epistemic modals—in particular, the fact that *all* the English strong necessity, weak necessity, and possibility modals exhibit this feature in their epistemic readings—has been appreciated only relatively recently.[22] This has likely made it look as though the evidential feature of English modals is less interesting and important than it is. I take it to be a high priority; indeed I think that a theory of English expressions of subjective uncertainty is incomplete without an account of the evidential feature of English epistemic modals. Such an account ought to answer two questions. How do epistemic modals signal that their prejacents are the conclusion of an inference? And why do they carry such a signal?

Following (but also somewhat extending) the work of Angelika Kratzer, I hold that whether or not they are read epistemically the modals 'must,' 'have to,' 'should,'

[21] Adding an explicit specification of the relevant information—like 'given what we know about when he left'—can help make the epistemic readings vivid.

[22] See Swanson 2006, 56 and von Fintel & Gillies 2010 for the evidential feature of epistemic possibility modals. See Huddleston & Pullum 2002, 186, Copley 2006, Swanson 2008, 1203–1205, and von Fintel & Gillies 2010 for the evidential feature of weak necessity modals. Karttunen does write that "…what is true about *must* is also true of other similar words, such as *necessarily…, have to,* or *is bound to*" (1972, 13), but he does not mention weak necessity modals or possibility modals.

'ought,' 'can,' 'could,' 'might,' and so on all require that the context supply a set of premises.[23] In the case of 'must' and 'have to,' it is (very roughly speaking) from these premises that the prejacent is said to follow. Epistemic 'must' and 'have to' carry an inferential signal because the Kratzerian dimension in their semantics requires a contextually supplied set of premises from which the prejacent is said to follow. In the case of 'can't' and 'couldn't,' the negation of the prejacent is said to follow from these premises. As with 'must' and 'have to', the semantic values of epistemic 'can' and 'could' look for a body of evidence that (roughly speaking) does not rule out the possibility that the prejacent is true. And so 'can' and 'could' also carry an inferential signal, which is easiest to see with the help of wide scope negation.

In more detail: epistemic modals are evaluated relative to a set of arguments, modeled as a set of consistent premises, or, equivalently, as a partial preorder over possible worlds (Lewis 1981). The arguments draw on a single body of information and, intuitively, are ordered by strength, where argument *A* is stronger than argument *B* iff the premises of argument *A* include all the premises in argument *B* and more besides. If neither argument includes all the premises of the other, then they are incomparable with respect to strength. If we make the limit assumption (Lewis 1973, 19–20) to guarantee that there are strongest (or tied for strongest) arguments, then on the Kratzer/Veltman semantics the following sentence says that 'ϕ' follows from all the strongest arguments available.[24]

(35) It must be/has to be that ϕ.

That is, (36) says that 'ϕ' follows from all the arguments that include as much available information as possible without lapsing into inconsistency. Given that 'might' is dual to 'must,' the following sentence says (to a first approximation) that the strongest arguments available do not falsify 'ϕ':

(36) It might be that ϕ.

What is important here is that when speakers use these modals, they generally presuppose that the context supplies a set of arguments that bear on the truth value of the relevant modal's prejacent. The evidential feature carried by English epistemic modals just is this presupposition.

Weak necessity modals like 'should' and 'ought' also have an evidential feature, as we saw earlier with examples like (32) and (33):

[23]For Kratzer's discussions see especially her 1976, 1977, and 1991. The view I sketch here differs somewhat from Kratzer's view. In particular, she treats the ordering source of epistemic modals as representing stereotypicality (1991, 644), whereas I treat it as representing premises taken to support belief in the prejacent, where 'things are proceeding normally' might be one of those premises.

[24]Without the limit assumption, I favor the view developed in Swanson 2011b.

(32) John should be here by now.

(33) John ought to be here by now.

I follow Lemmon 1962, Williams 1973, van Fraassen 1973, Marcus 1980 and the many others who reject semantics for weak necessity modals according to which these modals validate agglomeration. That is, I hold that it doesn't follow from the fact that it ought to be that ϕ and it ought to be that ψ that it ought to be that ϕ and ψ. I favor a semantics for weak necessity modals on which such modals bracket whatever incomparabilities there are in the relevant set of arguments, in effect asking whether there is some way of eliminating those incomparabilities that would make 'Must ϕ' true (Swanson 2011b, 705–708). (More formally, and in a Kratzerian framework: 'Ought ϕ' is true relative to a preordered set of arguments iff 'Must ϕ' is true relative to some maximal chain of that preordered set of arguments. Here a *preorder* is a reflexive and transitive binary relation; a *total* preorder admits no incomparabilities; a *chain* is a totally preordered subset of the set of arguments; and a chain is *maximal* iff it is not a subset of any other chain.) On this way of thinking, weak necessity modals inherit some important features from strong necessity modals—including their evidentiality.

While the quantitative aspects of the language of subjective uncertainty make constraint semantics look attractive, the evidential aspects of epistemic modals make premise semantics look attractive. For all I've said so far, it might seem that constraint semantics is not really needed for epistemic modals. But this line has a problem: pure premise semantics effaces important differences between strong and weak necessity modals in their epistemic readings. On their epistemic readings, (30) and (31) are anomalous though (32) and (33) are fine.

(30) #John must be here by now. But he's not here yet.

(31) #John has to be here by now. But he's not here yet.

(32) John should be here by now. But he's not here yet.

(33) John ought to be here by now. But he's not here yet.

So while both weak and strong necessity modals typically signal that their prejacent is the conclusion of an inference, only strong necessity epistemic modals, as in (30) and (31), convey that the speaker endorses any level of credal commitment to the prejacent. Epistemic readings of weak necessity modals are noncommittal with respect to the speaker's credence in the prejacent—indeed, they are very naturally used alongside the *denial* of the prejacent (Copley 2006; cf. Copley 2004).

This contradicts the surprisingly common view that epistemic readings of weak necessity modals involve commitment to the 'probable' truth of their prejacents (see, e.g., Sloman 1970, Horn 1972 and 1989, Wheeler 1974, and, in more recent work, Huddleston & Pullum 2002, Wedgwood 2006, Thomson 2008, and Finlay 2010). I do not see how one could sustain this view given the contrasts between (30)/(31) and (32)/(33), and between (32)/(33) and (37).

(37) #John is probably here by now. But he's not here yet.

Tellingly, Huddleston and Pullum give an example that suggests that the weak modal 'is supposed to' has a meaning like that I am defending here: they gloss the epistemic reading of 'It is supposed to have been posted yesterday' as 'It's alleged to have been posted yesterday' (208).

The most important point, for my purposes, is that epistemic readings of possibility modals are actually in some ways stronger than epistemic readings of weak necessity modals. For example, if 'might' is read epistemically then (38) is an "epistemic contradiction," in Seth Yalcin's nice phrase (2007, 984).

(38) #He left an hour ago, and there isn't any traffic. So John might be here by now, but he's not here yet.

Although the use of 'John might be here by now' certainly does not represent a speaker as lending *high* credence to the proposition that John is here by now, it does represent the speaker as lending *some* credence to that proposition. So epistemic possibility modals are stronger than epistemic weak necessity modals in that only the former commit the speaker to credence in the prejacent. Epistemic 'should' and 'ought' do not imply epistemic 'can,' 'could,' and 'might.'[25]

Pure premise semantics, on its own, wrongly has it that epistemic 'ought' and 'should' *do* imply epistemic 'can,' 'could,' and 'might.' But we can block this implication by supplementing pure premise semantics with constraint semantics. Consider

(39) John might be here by now.

It is straightforward to give a semantic entry for 'might' on which the constraint associated with this sentence involves both lending credence above some threshold to the proposition that John is here now *and* believing the proposition that truth-

[25]I don't have space here for serious discussion of the subtleties of Yalcin's data involving supposition of epistemic contradictions. But there may be reason to appeal to the credal constraints of strong necessity modals and possibility modals to help explain why the epistemic readings of (i) and (ii) are odd, even though (iii) is fine.

(i) #Suppose it is raining and it might not be raining. (Yalcin 2005, 234; 2007, 984)

(ii) #Suppose it is raining and it needn't be raining.

(iii) Suppose it is raining and it shouldn't be raining. (Then we'd need to recalibrate our instruments.)

conditional premise semantics associates with (39). Similarly, it is straightforward to give a semantic entry for 'must' on which the constraint associated with (40) requires high credence in the proposition that John is here now.

(40) John must be here by now.

These semantic entries would explain why English modals carry evidential presuppositions, and also explain the credal aspect of the strong necessity and possibility modals. No similar constraint should be written into the constraint semantic entries for weak necessity modals. This approach accounts for the oddness of (30), (31), and (38)—it is odd to advise one's addressee to have positive credence in a proposition while at the same time denying that proposition—without wrongly predicting that (32) and (33) will also be odd. For this reason I favor a theory of epistemic strong necessity modals and epistemic possibility modals that blends pure premise semantics and pure constraint semantics. We should address the inadequacies in the pure views by synthesizing them into a hybrid.

3. Conclusion

As I suggested at the beginning of this paper, the assertion hypothesis—the hypothesis that propositions suffice to characterize the contents of assertions—is supported by two influential thoughts. One is that the essential function of assertion is to represent the world as being a certain way; the other is that the successes of traditional truth-conditional semantics are hard or impossible to duplicate in non-truth-conditional frameworks. Because features of credal states that would be worth communicating are generally communicable, the first thought is undercut by the fact that we cannot individuate credal states themselves purely in terms of how they represent the world as being. Most of the work in this paper is aimed at undercutting the second thought. We can get compositionality with constraint semantics; truth-conditional semantics is not necessary. And an extremely influential part of the truth-conditional canon—premise semantics—has flaws that can be rectified by adding appropriate credal constraints to the semantic entries for strong necessity modals and possibility modals.

References

Beaney, Michael, editor. 1997. *The Frege Reader*. Blackwell Publishers, Ltd., Oxford.

Carnap, Rudolf. 1956. *Meaning and Necessity: A Study in Semantics and Modal Logic*. University of Chicago Press, Chicago, second edn.

Chapman, Siobhan. 2005. *Paul Grice: Philosopher and Linguist*. Palgrave Macmillan.

Coates, Jennifer. 1983. *The Semantics of the Modal Auxiliaries*. Croom Helm, London.

Copley, Bridget. 2004. "So-Called Epistemic *Should*." *Snippets*, vol. 9: 7–8.

—. 2006. "What Should *Should* Mean?" Ms., CNRS/Université Paris 8, URL `http://copley.free.fr/copley.should.pdf`.

Dowell, J. L. 2011. "A Flexible Contextualist Account of Epistemic Modals." *Philosophers' Imprint*, vol. 11 (14): 1–25.

Dummett, Michael. 1981. *Frege: Philosophy of Language*. Harvard University Press, Cambridge, second edn.

Egan, Andy & Brian Weatherson, editors. 2011. *Epistemic Modality*. Oxford University Press, Oxford.

Finlay, Stephen. 2010. "What *Ought* Probably Means, and Why You Can't Detach It." *Synthese*, vol. 177: 67–89.

von Fintel, Kai & Anthony S. Gillies. 2005. "'Might' Made Right." Available online at `http://mit.edu/fintel/www/might-austin.pdf`.

—. 2008. "CIA Leaks." *Philosophical Review*, vol. 117 (1): 77–98.

—. 2010. "*Must* … Stay … Strong!" *Natural Language Semantics*.

—. 2011. "'Might' Made Right." In Egan & Weatherson (2011).

Foley, Richard. 1992. "The Epistemology of Belief and the Epistemology of Degrees of Belief." *American Philosophical Quarterly*, vol. 29 (2): 111–124.

van Fraassen, Bas C. 1973. "Values and the Heart's Command." *Journal of Philosophy*, vol. 70 (1): 5–19.

—. 1980. "Rational Belief and Probability Kinematics." *Philosophy of Science*, vol. 47 (2): 165–187.

—. 1981. "A Problem for Relative Information Minimizers in Probability Kinematics." *British Journal for the Philosophy of Science*, vol. 32 (4): 375–379.

—. 1989. *Laws and Symmetry*. Oxford University Press, Oxford.

van Fraassen, Bas C., R. I. G. Hughes & Gilbert Harman. 1986. "A Problem for Relative Information Minimizers, Continued." *British Journal for the Philosophy of Science*, vol. 37 (4): 453–463.

Frege, Gottlob. 1892a. "On Concept and Object." In Beaney (1997), 181–193. Translated by Peter Geach.

—. 1892b. "On *Sinn* and *Bedeutung*." In Beaney (1997), 151–171. Translated by Max Black.

Gibbard, Allan. 1981. "Two Recent Theories of Conditionals." In *Ifs: Conditionals, Belief, Decision, Chance, and Time*, William L. Harper, Robert Stalnaker & Glenn Pearce, editors, 211–247. D. Reidel Publishing Company, Dordrecht.

—. 1990. *Wise Choices, Apt Feelings*. Harvard University Press, Cambridge.

—. 2003. *Thinking How to Live*. Harvard University Press, Cambridge.

Grice, Paul. 1987. "Logic and Conversation." In *Studies in the Way of Words*. Harvard University Press, Cambridge.

Heim, Irene. 1983. "On the Projection Problem for Presuppositions." In Portner & Partee (2002), 249–260.

—. 1991. "Articles and Definiteness." Ms., Department of Linguistics and Philosophy, MIT. Originally published in German as "Artikel und Definitheit," in von Stechow and Wunderlich (eds.) *Semantics: An International Handbook of Contemporary Research,* Berlin: de Gruyter, 1991.

Horn, Laurence R. 1972. *On the Semantic Properties of the Logical Operators in English*. Ph.D. thesis, UCLA.

—. 1989. *A Natural History of Negation*. University of Chicago Press, Chicago.

Huddleston, Rodney & Geoffrey K. Pullum, editors. 2002. *The Cambridge Grammar of the English Language*. Cambridge University Press, Cambridge.

Jaynes, E. T. 1957. "Information Theory and Statistical Mechanics." *The Physical Review*, vol. 106 (4): 620–630.

Jeffrey, Richard C. 1968. "Probable Knowledge." In *Probability and the Art of Judgment*, 30–43. Cambridge University Press, Cambridge.

—. 1983. "Bayesianism with a Human Face." In *Testing Scientific Theories*, John Earman, editor, vol. 10, 133–156. University of Minnesota Press, Minneapolis.

JOYCE, JAMES M. 2005. "How Probabilities Reflect Evidence." *Philosophical Perspectives*, vol. 19: 153–178.

KAMP, HANS. 1981. "A Theory of Truth and Semantic Representation." In PORTNER & PARTEE (2002), 189–222.

KAPLAN, MARK. 1996. *Decision Theory as Philosophy*. Cambridge University Press, Cambridge.

KARTTUNEN, LAURI. 1972. "*Possible* and *Must*." In *Syntax and Semantics*, J. KIMBALL, editor, vol. 1, 1–20. Academic Press, New York.

KRATZER, ANGELIKA. 1976. "Was 'Können' und 'Müssen' Bedeuten Können Müssen." *Linguistische Berichte*, vol. 42: 1–28.

—. 1977. "What *Must* and *Can* Must and Can Mean." *Linguistics and Philosophy*, vol. 1: 337–355.

—. 1991. "Modality." In *Semantics: An International Handbook of Contemporary Research*, ARNIM VON STECHOW & DIETER WUNDERLICH, editors, 639–650. W. de Gruyter, Berlin.

KULLBACK, SOLOMON & RICHARD A. LEIBLER. 1951. "On Information and Sufficiency." *Annals of Mathematical Statistics*, vol. 22 (1): 79–86.

LEMMON, E. J. 1962. "Moral Dilemmas." *Philosophical Review*, vol. 71 (2): 139–158.

LEVI, ISAAC. 1980. *The Enterprise of Knowledge*. MIT Press, Cambridge.

—. 1985. "Imprecision and Indeterminacy in Probability Judgment." *Philosophy of Science*, vol. 52 (3): 390–409.

LEWIS, DAVID K. 1973. *Counterfactuals*. Basil Blackwell Ltd., Malden, MA.

—. 1981. "Ordering Semantics and Premise Semantics for Counterfactuals." *Journal of Philosophical Logic*, vol. 10: 217–234.

MACFARLANE, JOHN. 2011. "Epistemic Modals Are Assessment-Sensitive." In EGAN & WEATHERSON (2011), 144–178.

MARCUS, RUTH BARCAN. 1980. "Moral Dilemmas and Consistency." *Journal of Philosophy*, vol. 77 (3): 121–136.

MONTAGUE, RICHARD. 1973. "The Proper Treatment of Quantification in Ordinary English." In PORTNER & PARTEE (2002), 17–34.

Moore, G. E. 1962. *The Commonplace Book of G. E. Moore: 1919–1953*. George Allen & Unwin Ltd.

Moss, Sarah. forthcoming. "On the Semantics and Pragmatics of Epistemic Vocabulary." *Semantics and Pragmatics*.

Palmer, F. R. 2001. *Mood and Modality*. Cambridge University Press, Cambridge, second edn.

Portner, Paul & Barbara H. Partee, editors. 2002. *Formal Semantics: The Essential Readings*. Blackwell Publishers Ltd., Oxford.

Quine, W. V. O. 1953. "Three Grades of Modal Involvement." In *The Ways of Paradox*, 158–176. Random House, New York.

Russell, Bertrand. 1903. *Principles of Mathematics*. Norton, New York, second edn.

Russert, Tim. 2006. *Wisdom of Our Fathers: Lessons and Letters from Daughters and Sons*. Random House, New York.

Schroeder, Mark. 2008. *Being For: Evaluating the Semantic Program of Expressivism*. Oxford University Press, Oxford.

Seidenfeld, Teddy. 1986. "Entropy and Uncertainty." *Philosophy of Science*, vol. 53 (4): 467–291.

Sloman, Aaron. 1970. "'Ought' and 'Better'." *Mind*, vol. 79: 385–394.

Speas, Peggy. 2008. "On the Syntax and Semantics of Evidentials." *Language and Linguistics Compass*, vol. 2 (5): 940–965. DOI: 10.1111/j.1749-B1Bx.200B.00069.x.

Stalnaker, Robert C. 1978. "Assertion." In *Context and Content*, 78–95. Oxford University Press, Oxford.

—. 1984. *Inquiry*. MIT Press, Cambridge.

Sturgeon, Scott. 2008. "Reason and the Grain of Belief." *Noûs*, vol. 42: 139–165.

Swanson, Eric. 2006. *Interactions with Context*. Ph.D. thesis, Massachusetts Institute of Technology.

—. 2008. "Modality in Language." *Philosophy Compass*, vol. 3 (6): 1193–1207. DOI: 10.1111/j.1747-9991.2008.00177.x.

—. 2010a. "On Scope Relations between Quantifiers and Epistemic Modals." *Journal of Semantics*, vol. 27 (4): 529–540.

—. 2010b. "Structurally Defined Alternatives and Lexicalizations of XOR." *Linguistics and Philosophy*, vol. 33 (1): 31–36.

—. 2011a. "How Not to Theorize about the Language of Subjective Uncertainty." In Egan & Weatherson (2011).

—. 2011b. "On the Treatment of Incomparability in Ordering Semantics and Premise Semantics." *Journal of Philosophical Logic*, vol. 40 (6): 693–713.

—. 2012. "Propositional Attitudes." In *Semantics: An International Handbook of Natural Language Meaning*, Claudia Maienborn, Klaus von Heusinger & Paul Portner, editors, vol. 3, 1538–1561. Mouton de Gruyter, Berlin.

Thomason, Richmond H. 1969. "Modal Logic and Metaphysics." In *The Logical Way of Doing Things*, Karel Lambert, editor, 119–146. Yale University Press, New Haven.

Thomason, Richmond H. & Robert C. Stalnaker. 1968. "Modality and Reference." *Noûs*, vol. 2 (4): 359–372.

Thomson, Judith Jarvis. 2008. *Normativity*. Open Court, Chicago.

Walley, Peter. 1991. *Statistical Reasoning with Imprecise Probabilities*. Chapman and Hall, New York.

Wedgwood, Ralph. 2006. "The Meaning of 'Ought'." *Oxford Studies in Metaethics*, vol. 1: 127–160.

Wheeler, Samuel C., III. 1974. "Inference and the Logical "Ought"." *Noûs*, vol. 8: 233–258.

Williams, Bernard. 1973. "Ethical Consistency." In *Problems of the Self: Philosophical Papers, 1956–1972*, 166–186. Cambridge University Press, Cambridge.

Williams, Peter M. 1980. "Bayesian Conditionalization and the Principle of Minimum Information." *British Journal for the Philosophy of Science*, vol. 31 (2): 131–144.

Williamson, Timothy. 2000. *Knowledge and its Limits*. Oxford University Press, Oxford.

Yalcin, Seth. 2005. "Epistemic Modals." In *New Work on Modality*, Jon Gajewski, Valentine Hacquard, Bernard Nickel & Seth Yalcin, editors, 231–272. MIT Working Papers in Linguistics, vol. 51.

—. 2007. "Epistemic Modals." *Mind*, vol. 116: 983–1026.

—. 2010. "Probability Operators." *Philosophy Compass*, vol. 5 (11): 916–937.

—. 2011. "Nonfactualism about Epistemic Modality." In Egan & Weatherson (2011).